Exact asymptotic behaviour of deterministic and stochastic differential and difference equations with regularly varying coefficients

John A. D. Appleby
School of Mathematical Sciences, Dublin City University, Ireland

In this talk, we analyse the exact speed of convergence of solutions to perturbed differential systems to the steady state of an unperturbed differential equation. The perturbations may be either deterministic or stochastic in nature. The equations considered are intrinsically nonlinear, in the sense that the leading order of any state–dependent restoring term is not linear at the equilibrium. The equations considered have regularly varying nonlinearities at the equilibrium, which can lead to sub–polynomial, power–like, sub–exponential and super–exponential rates of decay to the equilibrium. Roughly speaking, a critical rate of decay of the perturbation can be identified such that perturbations decaying more rapidly than this critical rate lead to convergence rates of solutions which are the same as those of the unperturbed equation, while slower decaying perturbations lead to identifiable and slower convergence rates of the solution. Extensions to difference equations (viewed as discretisations of the underlying continuous–time equations) as well as functional differential equations, are also sketched.

Stability in time-delayed differential equations from hematopoietic models

Jacques Bélair
Université de Montréal, Canada

Time-delayed differential equations occur naturally in the representation of population models in general, and of stage-structured cell populations in particular. We present progressively more sophisticated models for hematopoiesis (production of blood cells) and analyse the stability of their equilibrium solutions. Characteristic equations obtained by linearisation around these equilibria will be shown to naturally contain multiple delays as well as linearised state-dependant delays as a consequence of well-identified physiological hypotheses. The complexity of the characteristic equations and the associated stability charts, and bifurcation diagramms, will be shown *not* to be a monotonically increasing function of the complexity of the models.

Supported by NSERC (Canada) and FRQNT (Québec).

On stability of equations with a distributed delay

Elena Braverman
University of Calgary, Canada

We study delay-independent stability in nonlinear models with a distributed delay and one or several positive equilibrium points which occur in population dynamics and other applications. It is assumed that the distributed delay is incorporated into the production term only. In particular, for models with one positive equilibrium we construct a relevant difference equation such that its stability implies stability of the equation with a distributed delay and a finite memory. This result is, generally speaking, incorrect for systems with infinite memory. If the relevant difference equation is unstable, we describe the general delay-independent lower and upper solution bounds and also demonstrate that the equation with a distributed delay is stable for small enough delays.

In the case when the production function incorporating the delay is monotone increasing, the qualitative behaviour of such equations can be comprehensively described also in the case of multiple positive equilibrium points. This is a joint work with Leonid Berezansky (Ben Gurion University of the Negev, Israel) and Sergey Zhukovskiy (Moscow PF University, Russia).

On delay-dependent stability conditions for a three-term linear difference equation

Jan Čermák
Institute of Mathematics, Brno University of Technology, Czech Republic

The problem of necessary and sufficient conditions for the asymptotic stability of the three-term linear difference equation

$$y(n+k)+\alpha y(n)+\beta y(n-\ell)=0\,,\qquad \alpha,\,\beta\in\mathbb{R},\quad k,\,\ell\in\mathbb{Z}^+ \tag{1}$$

first appeared in 1976, when A. Levin and R.M. May presented the stability criterion for (1) with $k=1$ and $\alpha=-1$. Since the time, many outstanding mathematicians (such as S. Elaydi, I. Győri, M. Kipnis, S. Kuruklis, G. Ladas and others) significantly contributed to this topic and formulated various types of stability conditions for (1) and its particular cases.

In this contribution, we give a survey of the most relevant results and present a new type of necessary and sufficient conditions guaranteing the asymptotic stability of (1). Contrary to the existing ones, these conditions are given explicitly in terms of parameters α, β, k, ℓ and form a discrete analogue to stability conditions for underlying delay differential equations.

Positive solutions of delayed differential equations

Josef Diblík
Brno University of Technology, Czech Republik

Using a comparison with the integro-differential equation

$$\dot{y}(t) + \int_{t_0}^{t} e^{-\int_s^t a(\xi)d\xi} b(s)y(h(s))ds = 0,$$

the existence of positive solutions is studied for the second-order delay differential equation with a damping term

$$\ddot{x}(t) + a(t)\dot{x}(t) + b(t)x(h(t)) = 0.$$

Comparison type results and explicit non-oscillation criteria are derived. The talk is based on joint results with L. Berezansky and Z. Šmarda.

Stabilization of second order delay differential equations

Alexander Domoshnitsky
Ariel University, Israel

In this talk a problem of stabilization is considered. New sufficient conditions of nonoscillation and stability of the second order delay differential equations without damping term are proposed. Examples on essentiality of these conditions are discussed. A special technique of differential inequalities is developed. The results are based on assertions about positivity of the Cauchy functions and entries of corresponding Cauchy matrices.

Positive solutions of super-linear Emden-Fowler differential equation

Zuzana Došlá
Masaryk University Brno, Czech Republic

We study the second-order Emden-Fowler type differential equation

$$(a(t)|x'|^{\alpha}\operatorname{sgn} x')' + b(t)|x|^{\beta}\operatorname{sgn} x = 0, \tag{1}$$

in the super-linear case $0 < \alpha < \beta$. Using a Hölder-type inequality, we resolve the open problem on the possibile coexistence on three types of nonoscillatory solutions (subdominant, intermediate and dominant solutions). Jointly with this, sufficient conditions for the existence of globally positive intermediate solutions are established.

This problem has a long history. For Emden-Fowler equation

$$x'' + b(t)|x|^{\beta}\operatorname{sgn} x = 0, \quad \beta \neq 1, \tag{2}$$

it started sixty years ago by Atkinson, Moore-Nehari in case $\beta > 1$ and Belohorec in case $\beta < 1$. In particular, it was proved that this triple coexistence is impossible.

For the more general equation (1), this study was continued in nineties of the last century. Recently, the question of the possible triple coexistence has been resolved in negative way by Naito in the sublinear case $\alpha > \beta > 0$.

This is a joint work with Mauro Marini (University of Florence).

Modified Riccati technique in the half-linear oscillation theory

Ondřej Došlý
Masaryk University Brno, Czech Republic

We consider the second-order half-linear differential equation

$$L(x) := (r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) = |x|^{p-2}x, \ p > 1, \tag{1}$$

with continuous functions r, c and $r(t) > 0$. The classical Riccati technique consists in the relationship between (1) and the Riccati type equation

$$w' + c(t) + (p-1)r^{1-q}(t)|w|^q = 0, \quad q = \frac{p}{p-1}, \tag{2}$$

related to (1) by the Riccati substitution $w = r\Phi(x'/x)$. The modified Riccati equation is the equation

$$v' + hL(h) + (p-1)r^{1-q}h^{-q}[|v+G|^q - q\Phi^{-1}(G)v - G^q] = 0,$$

with $\Phi^{-1}(s) = |s|^{q-2}s$ and $G = rh\Phi(h')$, which results from (2) upon the transformation $v = h^p(w - w_h)$, $w_h = r\Phi(h'/h)$. It turns out the the modified Riccati technique can be applied in situations when the classical technique does not give satisfactory results. We will report some recent results along this line.

Global attractivity and extinction for Lotka-Volterra systems with infinite delay and feedback controls

Teresa Faria
University of Lisbon, Portugal

We consider a multiple species Lotka-Volterra model with infinite distributed delays and feedback controls, for which we assume a form of diagonal dominance of the instantaneous negative intra-specific terms over the infinite delay effect in both the population variables and controls. General sufficient conditions for the existence and attractivity of a saturated equilibrium are established. When the saturated equilibrium is on the boundary of $\mathbb{R}^n_+$, sharper criteria for the extinction of all or part of the populations are given. While the literature usually treats the case of competitive systems only, here no restrictions on the signs of the intra- and inter-specific delayed terms are imposed.

This is a joint work with Prof. Yoshiaki Muroya, from Waseda University, Japan.

Positive steady states of some nonlinear evolution equations

Jozsef Z. Farkas
University of Stirling, Scotland

We study the question of existence of positive steady states of nonlinear evolution equations. We recast the steady state problem as an eigenvalue problem for a parameterised family of unbounded linear operators, which are generators of strongly continuous semigroups; and a fixed-point problem. In case of irreducible semigroups we consider evolution equations with non-monotone nonlinearities of dimension 2, and we establish some new fixed-point theorems for set valued maps. In case of reducible semigroups we establish results for monotone nonlinearities of any finite dimension n. In addition we establish a non-quasinilpotency result for a class of strictly positive operators, which are neither irreducible nor compact, in general. We illustrate our theoretical results using examples of partial differential equations arising in structured population dynamics.

This is joint work with Angel Calsina, Universitat Autonoma de Barcelona.

On periodic solutions of a ring network

Barnabás M. Garay
Pázmány Péter Catholic University, Budapest, Hungary

A ring of $N = 2M$ identical neuron cells with piecewise linear and saturated bidirectional coupling nonlinearities is considered. For certain values of the two coupling parameters, existence of hyperbolic periodic solutions with cyclic symmetry is established. With $M \to \infty$, the dominant Floquet multiplier converges to 1 and the remaining $2M - 2$ nontrivial Floquet multipliers converge to 0. In both cases, the convergence is proved to be exponential. Waveform asymptotics as well as the asymptotics of the dominant eigenvector are also presented.

The entire work was motivated by electrical circuit experiments.

The talk is based on ongoing research with *Mauro Forti, Miklós Koller*, and *Luca Pancioni.*

Two approximating techniques for delay differential equations with application

István Győri
University of Pannonia, Hungary

Our lecture is focussing on two approximation techniques and their applications in mathematical biology. In both cases, we establish connections between the solutions of a delay differential equation and of a suitably constructed n dimensional system of ordinary differential equations. The two approximating schemes are originated from two different ideas. The first one is based on the connection between transit compartments models which is frequently used in pharmacology and the theory of compartmental models with pipes. The second scheme is derived from a relation between delay differential equations and hyperbolic type partial differential equations which is combined with the method of lines approximation.

In both cases the convergence is uniform on any compact interval as the dimension n tends to infinity. It is worth to note that in the first approximation case the convergence is also uniform on $[0, \infty)$, under some extra condition.

Analysis of qualitative dynamic properties of positive polynomial systems using transformations

Katalin M. Hangos*
University of Pannonia, Hungary

Gábor Szederkényi
Peter Pázmány Catholic University, Hungary

Two classes of positive polynomial systems, quasi-polynomial (QP) systems and reaction kinetic networks with mass action law (MAL-CRN) are considered. QP-systems are general descriptors of ODEs with smooth right-hand sides, their stability properties can be checked by algebraic methods (linear matrix inequalities). On the other hand, MAL-CRN systems possess a combinatorial characterization of their structural stability properties using their reaction graph.

Dynamic equivalence and similarity transformations applied to either the variables (quasi-monomial and time-reparametrization transformations) or the phase state space (translated X-factorable transformation) will be applied to construct a dynamically similar linear MAL-CRN model to certain given QP system models. This way one can establish sufficient structural stability conditions based on the underlying reaction graph properties for the subset of QP system models that enable such a construction.

Existence of periodic solutions in linear higher order system of difference equations

László Horváth
University of Pannonia, Hungary

In this talk we investigate the existence of nontrivial periodic solutions of a higher order system of difference equations. The results are based on earlier periodicity conditions by István Győri and László Horváth. We also use the theory of circulant matrices combined by a theorem of Silvester on the computation the determinant of block matrices. An illustrative application is given to show the effectiveness of of our framework and to point out the connection between our periodicity results and some known stability conditions.

Travelling around obstacles in planar anistropic spatial systems

H.J. Hupkes*
Leiden University, The Netherlands

A. Hoffman
Olin College, MA, USA

E. Van Vleck
U. Kansas, KS, USA

We study dynamical systems posed on a discrete spatial domain, with a special focus on the behaviour of basic objects such as travelling waves under (potentially large) perturbations of the wave and the underlying spa- tial lattice. Such travelling waves satisfy functional differential equations of mixed type, which can be seen as generalizations of delay equations in the sense that both delays and advances in the arguments are allowed.

Convergence of solutions of impulsive delay differential equations

Fatma Karakoç* and Hüseyin Bereketoglu
Ankara University, Turkey

István Győri
University of Pannonia, Hungary

In this paper, we consider a nonhomogeneous linear impulsive delay differential equations system

$$x'(t) = A_0(t)x(t) + \sum_{l=1}^{L} A_l(t)x(t-\tau_l) + f(t), \qquad t \geq t_0,\ t \neq \theta_i,$$

$$\Delta x(\theta_i) = B_0(i)x(\theta_i) + \sum_{k=1}^{K} B_k(i)x(\theta_{i-m_k}) + g(i), \qquad i \in Z^+.$$

Sufficients conditions are obtained for the convergence of solutions. Moreover, the limit of the solution is formulated.

Symmetric periodic solutions for a class of differential delay equations with distributed delay

Benjamin Kennedy
Gettysburg College, USA

We consider the nonlinear distributed delay equation

$$x'(t) = f\left[\int_{t-1}^{t-d} g(x(s))\ ds\right], \quad d \in [0,1),$$

where f has negative feedback and g has positive feedback.

We describe conditions on f and g (including oddness and monotonicity) under which the above equation has a periodic solution of period $2+2d$ that satisfies certain symmetries.

We also describe the dynamics of certain related "model" equations where f or g (or both) are step functions.

On the boundedness of positive solutions of the reciprocal max-type difference equation $x_n = \max\left\{\frac{A_n^1}{x_{n-1}}, \frac{A_n^2}{x_{n-2}}, \dots, \frac{A_n^t}{x_{n-t}}\right\}$ with periodic parameters

Daniel W. Cranston and Candace M. Kent*
Virginia Commonwealth University, USA

We investigate the boundedness of positive solutions of the reciprocal max-type difference equation

$$x_n = \max\left\{\frac{A_n^1}{x_{n-1}}, \frac{A_n^2}{x_{n-2}}, \dots, \frac{A_n^t}{x_{n-t}}\right\}, \qquad n = 1, 2, \dots,$$

where, for each value of i, the sequence $\{A_n^i\}_{n=0}^{\infty}$ of positive real numbers is periodic with period p_i. We give both sufficient conditions on the p_i's for the boundedness of all solutions and sufficient conditions for all solutions to be unbounded. This work essentially complements the work of Biddwell and Franke (2008), who showed that as long as every positive solution of our equation is *bounded*, then every positive solution is eventually periodic, thereby leaving open the question as to when solutions are bounded. We also briefly discuss the potential applications of this work to the biological area of morphogenesis.

Construction of invariant manifolds containing homoclinic orbits for delayed negative feedback

Tibor Krisztin
Bolyai Institute, University of Szeged, Hungary

We consider the delay differential equation $\dot{x}(t) = f(x(t), x(t-1))$ with a smooth $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying $f(0,0) = 0$ and $D_2 f < 0$. X.-B. Lin's equation (1987)

$$\dot{x}(t) = \frac{1}{\sinh 1}[(\cosh 1)x(t) - (1 + x^2(t))x(t-1)],$$

and the El Nino equation

$$\dot{x}(t) = \tau[x(t) - x^3(t) - \alpha x(t-1)]$$

of Suarez and Schopf (1988) are examples for delayed monotone negative feedback where homoclinic orbits may appear. We study the problem whether there are finite dimensional invariant manifolds containing the homoclinic orbits. It turns out that the global attractor of slow oscillation is a 2-dimensional invariant manifold containing the homoclinic orbits. The proof uses a slightly modified definition of slow oscillation since more stationary points may appear, moreover new types of return maps are applied.

Global attraction and global bifurcation in a system of delayed neural networks

Eduardo Liz
Universidad de Vigo, Spain

We establish a link between a system of functional differential equations and a finite-dimensional discrete dynamical system via a new notion of strong attractor. In some cases, studying the stability properties of an equilibrium for a one-dimensional map is enough to get sharp results of global attraction in systems of delay-differential equations.

Our main application is a system of delayed neural networks of Hopfield type, for which we provide results of global attractivity, multistability, and bifurcations. We point out that in general we do not assume monotonicity conditions in the activation functions, and our abstract setting allows us to deal with equations with distributed delay and neural systems with variable coefficients as well.

This talk is based on a joint work with Dr. Alfonso Ruiz Herrera (Universidad de Granada, Spain).

Asymptotically polynomial solutions of discrete Volterra equations

Małgorzata Migda
Institute of Mathematics, Poznań University of Technology, Poland

We consider the nonlinear discrete Volterra equations

$$\Delta^m x(n) = b(n) + \sum_{i=1}^{n} K(n,i) f\left(i, x(i)\right), \quad n \geq 1,$$

where $b : N \to R$, $K : N \times N \to R$ and $f : N \times R \to R$. We present sufficient conditions for the existence of asymptotically polynomial and asymptotically periodic solutions of studied equations. We also give conditions under which all solutions are asymptotically polynomial. The first order equations are studied separately.

Uniform persistence for monotone skew-product semiflows with applications

Rafael Obaya
University of Valladolid, Spain

In this talk we study some relevant dynamical properties of monotone and continuous skew-product semiflows. We consider the existence of a minimal set and introduce a convenient concept of uniform persistence for the order. We investigate both qualitative and quantitative characteristic which prove the presence of this kind of uniform persistence above and below the minimal set. The main results of the theory are given in terms of the principal spectrum of the linearized semiflow. We provide a method of calculus of the upper Lyapunov exponent that can be numerically implemented in applied models with practical interest described by functional differential equations with finite delay. This is a join work with Sylvia Novo and Ana M. Sanz.

Rigorous connecting orbits from numerics

Ken Palmer*
Providence University, Taiwan

Brian Coomes and Hüseyin Koçak
University of Miami, USA

A rigorous numerical method for establishing the existence of an orbit connecting two hyperbolic equilibria of a parametrized autonomous system of ordinary differential equations is presented. Given a suitable approximate connecting orbit and assuming that a certain associated linear operator is invertible, the existence of a true connecting orbit near the approximate orbit and for a nearby parameter value is proved. It turns out that inversion of the operator is equivalent to the solution of a boundary value problem for a nonautonomous inhomogeneous linear difference equation. A numerical procedure is given to verify the invertibility of the operator and obtain a rigorous upper bound for the norm of its inverse.

On well-posedness of neural field equations with delay

Arcady Ponosov*, Evgenii Burlakov, John Wyller
Norwegian University of Life Sciences, Norway

Evgenii Zhukovsky
Tambov State University, Russia

We study the convolution-like equation with a spatiotemporal delay

$$u(t,x) = \int_{-\infty}^{t} \eta(t,s) \int_{R^n} \omega(x,y) f(u(s-\tau(s,x,y),y))dyds, \tag{1}$$

which describes nonlocal heterogeneous neural field networks.

Here $u(t,x)$ denotes the synaptic input to a neural element at position x and time t, a sigmoidal function $f(u)$ gives the firing rate of a neuron with input u, the nonhomogeneous connectivity function $\omega(x,y)$ measures the strength of connections between neurons at positions x and y, $\eta(t,s)$ is the memory function representing synaptic processing of signals within the network, $\tau(s,x,y)$ describes the axonal delay effect arising from the finite speed of signal propagation between points x and y.

We define the notions of local, maximally extended and global solutions to equation (1) and obtain conditions for existence of a unique global or maximally extended solution and continuous dependence of solutions on the memory, connectivity, delay and initial functions.

Stability and control of systems with propagation

Vladimir Răsvan
University of Craiova, Romania

A natural way of introducing time delay equations is to consider boundary value problems for hyperbolic partial differential equations in two variables. Such problems account for the so called propagation phenomena that may be found in several physical and engineering applications.

Association of some functional equations to the aforementioned boundary value problems represents a way of tackling basic theory but also stability and feedback control. In this presentation there are considered some representative applications and their analysis is performed along the lines just described.

In developing the Liapunov method for such applications, the energy integral is used to generate suitable Liapunov functionals allowing synthesis of the stabilizing control as well as obtaining stability results.

Asymptotic behavior of extreme solutions to nonlinear differential systems in the framework of regular variation

Pavel Řehák
Academy of Sciences, Czech Republic

We consider the nonlinear differential system (denoted here by (S)) $x_i' = \delta a_i(t)F_i(x_{i+1})$, $i = 1,\dots,n,\ \ t \in [a,\infty)$, where x_{n+1} means x_1, $\delta \in \{-1,1\}$, $n \geq 2$, a_i are continuous regularly varying (at infinity) functions, and F_i are continuous functions with $|F_i(|\cdot|)|$ being regularly varying (at infinity or at zero) with a positive index. The subhomogeneity condition is assumed. System (S) includes several extensively studied objects: For instance, the n-th order two-term nonlinear equation $x^{(n)} = p(t)|x|^\beta \operatorname{sgn} x$, or some of its generalized variants, or the second order system which arises out when studying positive radial solutions to certain partial differential systems, or, after a certain modification, equations with a generalized Laplacian. We study asymptotic behavior of extreme solutions to system (S). By the extreme solutions we mean the so called strongly decreasing resp. strongly increasing solutions, that is, eventually positive solutions whose all components tend to zero (for (S) with $\delta = -1$) resp. to infinity (for (S) with $\delta = 1$) as $t \to \infty$. We show that — under quite natural conditions — such solutions exist and are (all) regularly varying with known index. Moreover, we establish precise asymptotic formulas. The results improve and extend some known results in various aspects. They are new even in some well studied special settings.

This is a joint work with Serena Matucci (University of Florence).

Asymptotic equivalence of impulsive differential equations in Banach space

Andrejs Reinfelds
Institute of Mathematics and Computer Science,
University of Latvia, Latvia

We consider nonperturbed and perturbed system of impulsive differential equations in Banach space. Using nonexponential Green type map we find sufficient conditions of asymptotic equivalence for systems of impulsive differential equations.

This work was partiallly supported by the grant Nr. 345/2012 of the Latvian Council of Science

Endemic bubbles generated by delayed behavioral response in epidemic models

Gergely Röst
University of Szeged, Hungary

Several models have been proposed to capture the phenomenon that individuals modify their behavior during an epidemic outbreak. This can be due to directly experiencing the rising number of infections, media coverage, or intervention policies. In this talk we show that a delayed activation of such a response can lead to some interesting dynamics.

For an SIS type process, if the delayed response occurs with a jump in the contact rate when the density of infection reaches some threshold, we show that for some interval of reproduction numbers, the system is oscillatory. The oscillation frequency is a discrete Lyapunov functional, and there exists a unique slowly oscillatory periodic solution with strong attractivity properties. We also construct rapidly oscillatory periodic solution of any frequency.

In the case of continuously decreasing transmission rate, if the response is not too sharp, the system preserves global stability. However, for sharp delayed response, we can observe stability switches as the basic reproduction number is increasing. First, the stability is passed from the disease free equilibrium to an endemic equilibrium via transcritical bifurcation as usual, but a further increase of the reproduction number causes oscillations, which later disappear, forming a structure in the bifurcation diagram what we call endemic bubble.

Joint work with Maoxing Liu, Eduardo Liz, Gabriella Vas.

Criteria for the existence of almost oscillatory solutions of second order neutral difference equations

Ewa Schmeidel* and Robert Jankowski
Institute of Mathematics, University of Białystok, Poland

Some new oscillation criteria for the neutral type difference equation

$$\Delta((\Delta(x_n + c_n x_{n-k}))^\gamma) + q_n x_{n+1}^\alpha + e_n sgn(x_n) = 0, \qquad n \in \mathbb{N}$$

are established by Riccati transformation techniques. The results are illustrated by examples.

Periodic orbits in differential equations with a large delay

Jan Sieber
University of Exeter, United Kingdom

Differential equations with a large delay in one of their arguments show features reminiscent of spatially extended systems. Eigenvalues of equilibria and periodic orbits tend to form bands, for which one can derive easily computable formulas. This permits us to draw conclusions for a system with given coefficients but increasing delay. For example, one can find criteria, which ensure the co-existence of large numbers of stable periodic orbits for large delays.

Dynamical systems approach to bone remodeling, neural networks and fluid dynamics

Stefan Siegmund
TU Dresden, Germany

In this talk we present three new ideas: (i) developing a bifurcation theory for a class of infinitely many differential equations and apply it to the remodeling of bone, (ii) combining graph theory and dynamical systems to study phase-locking of neurons, (iii) extending invariant manifold and spectral theory to fluid flows which are given by a finite amount of data.

On semilinear hyperbolic functional equations with state-dependent delays

László Simon
Institute of Mathematics, Eötvös Loránd University, Hungary

We shall consider weak solutions of initial-boundary value problems of the form

$$u''(t) + \tilde{Q}(u(t)) + \varphi(x)h'(u(t)) + H(t, x; u, u(\gamma_1(u)))+$$

$$G(t, x; u, u(\gamma_2(u)), u') = F, \quad t > 0, \quad x \in \Omega,$$

$$u(0) = u_0, \quad u'(0) = u_1$$

where $\Omega \subset R^n$ is a bounded domain and we use the notations $u(t) = u(t, x)$, $u' = D_t u$, $u" = D_t^2 u$, $\tilde{Q}$ is a linear second order symmetric elliptic differential operator in the variable x; h is a C^1 function having certain polynomial growth, H and G contain nonlinear functional (non-local) dependence on u, $u(\gamma_j(u))$ and u' with some polynomial growth. The delays $\gamma_j(u)$ are given by continuous (nonlinear) operators $\gamma_j : L^2((0, T) \times \Omega) \to C_a[0, T]$ satisfying $0 \le [\gamma_j(u)](t) \le t$, $[\gamma_j(u)]'(t) \ge c_0$ with some constant $c_0 > 0$.

Existence of solutions for $t \in (0, T)$ and $t \in (0, \infty)$ will be shown, further, uniqueness and some qualitative properties of the solutions will be proved. Finally, examples will be considered.

Bifurcations in a dynamical system model of epidemic propagation on adaptive networks

Péter L. Simon
Institute of Mathematics, Eötvös Loránd University Budapest, Hungary

Epidemic propagation is studied on an adaptive network. The links are activated and deleted according to certain rules modeling the phenomenon that susceptible nodes in the network try to cut their links to infected ones in order to avoid infection, while looking for new non-infected neighbours to keep their average degree at a certain level. The behaviour of the model is explored via numerical simulation and by using an approximating system of ODEs. Unlike on a static network, the resulting spectrum of behaviour is more complex with the number of infecting nodes exhibiting not only a single steady-state but also bistability and oscillations. It is shown that a stable endemic steady state can appear through transcritical bifurcation, or a stable and an unstable endemic steady state arise as a result of saddle-node bifurcation. Moreover, at the endemic steady state Hopf bifurcation may occur giving rise to stable oscillation.

The geometric description of a new global attractor

Gabriella Vas
MTA-SZTE Analysis and Stochastics Research Group, Hungary

This talk studies the delay equation $\dot{x}(t) = -\mu x(t) + f(x(t-1))$, where $\mu > 0$ and f is a strictly increasing, continuously differentiable nonlinear function with $f(0) = 0$ (positive feedback case).

According to previous results of Krisztin, Walther and Wu in the field, the global attractor –under mild technical conditions– contains three-dimensional spindle-like structures. The question, whether the global attractor is only the union of these structures, arose in the monograph of Krisztin, Walther and Wu. Krisztin and Vas have recently shown that this is not necessarily the case; there exists a feedback function f for which the equation has two so called large-amplitude periodic orbits outside the spindles.

As the Krisztin-Walther-Wu monograph contains the geometric and topologic description of the spindles, the need for a similar characterization of the rest of the global attractor comes up naturally. The purpose of the talk is to give such a description for the above mentioned feedback function.

Joint work with Tibor Krisztin.

Error bounds for solutions of nonlinear systems of Volterra integro-differential equations

Barbara Zubik-Kowal
Department of Mathematics, Boise State University, USA

In this talk, we will discuss approximate solutions of nonlinear and large systems of strongly joint Volterra integro-differential equations that model thalamo-cortical networks. To obtain the solutions of the systems at any time t, the behavior at all past times $0 \leq s \leq t$ has to be incorporated during computations and long-time intervals are considered. After introducing numerical schemes, we will focus on the analysis of the errors of the approximate solutions and derive error bounds showing their convergence to the exact solutions of general nonlinear systems. For thalamo-cortical systems with certain kernels applied in the sciences, we obtain stronger error bounds. The theoretical results validate that the numerical computations produce reliable solutions.

Integro-differential equations from biology with non-local consumption of resources

Narcisa Apreutesei
Department of Mathematics, "Gheorghe Asachi" Technical University of Iasi, Romania

We present some models of integro-differential equations from population dynamics, where the integral term describes the nonlocal consumption of resources. Fredholm property of the corresponding linear operators are useful to prove the existence of travelling wave solutions. For some models, this can be done only when the support of the integral is sufficiently small. In this case, the integro-differential operator is close to the differential one. One uses a perturbation method which combines the Fredholm property of the linearized operators and the implicit function theorem. For some other models, Leray-Schauder method can be applied. This implies the construction of a topological degree for the corresponding operators and the establishment of a priori estimates for the solution.

Boundedness of the solutions of delay Volterra difference equations

Essam Awwad
Department of Mathematics, Faculty of Science, Benha University, (P.O. 13518) - Benha, Egypt
Department of Mathematics, Faculty of Information Technology, University of Pannonia, Veszprém, Hungary

In this work we investigate the boundedness of the solutions of Volterra difference equations with time delay. We show some results in the critical case when the solutions of Volterra difference equations with time delay are bounded. We also apply our results on Volterra difference equations to prove the BIBO (bounded input bounded output) stability of nonlinear discrete control system with time delay.

This is a joint work with István Győri and Ferenc Hartung, University of Pannonia, Veszprém, Hungary.

A differential equation model of optimization of data transfer rate

István Balázs
Bolyai Institute, University of Szeged, Hungary

We consider a system of differential equations that has a delay depending on the solution. The time delay is defined by an ordinary differential equation with a non-continuous right hand side. The problem emerges in optimization (in the function of utility and price) of data transfer rate of computer networks.

The equation can not be inserted neither in the standard theory of functional differential equations nor in the theory of equations with state-dependent delay emerging in the recent years. Two main technical problems cause the difficulty: the state-dependent delay and the not smooth member in the algebraic equation defining the delay.

The main result is that the system defines a continuous semi-dynamical system. Under certain conditions we verify global convergence to the optimum (which is an equilibrium). We also prove results for existence of periodic solutions around the optimum.

New results: construction of appropriate phase space for the problem, verifying existence and uniqueness of solution in the phase space and continuous dependence on initial data, furthermore showing possibility of periodic behavior.

Equations with state-dependent delay in population biology

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A novel class of state-dependent delay equations is derived from the balance laws of age-structured population dynamics, assuming that birth rates and death rates, as functions of age, are piece-wise constant and that the length of the juvenile phase depends on the total adult population size. The resulting class of equations includes also neutral delay equations.

These equations can be written as systems for two variables consisting of an ordinary differential equation (ODE) and a generalized shift, a form suitable for numerical calculations. It is shown that the neutral equation (and the corresponding ODE - shift system) is a limiting case of a system of two standard delay equations.

We provide results on existence, uniqueness of solutions and linearized stability of equilibria, as well as numerical simulations.

Global stability of some second order difference equations

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Consider the difference equation

$$x_{k+1} = x_k e^{\alpha - x_{k-d}}$$

where α is a positive parameter and d is a nonnegative integer. The case $d = 0$ was introduced by W.E. Ricker in 1954. For the delayed version $d \geq 1$ of the equation S. Levin and R. May conjectured in 1976 that local stability of the nontrivial equilibrium implies its global stability. Based on rigorous, computer-aided calculations and analytical tools, we prove the conjecture for $d = 1$. We also apply our method to give necessary and sufficient conditions for the global stability of the trivial equilibrium of the difference equation $x_{k+1} = mx_k + \alpha \tanh x_{k-1}$, where m and α are real parameters. Joint work with Ferenc Bartha and Tibor Krisztin.

This research was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP 4.2.4. A/2-11-1-2012-0001 'National Excellence Program'.

Traveling fronts in KPP-Fisher equation

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We consider the semi-wavefronts (i.e. bounded solutions $u = \phi(x \cdot \nu + ct) > 0$, $|\nu| = 1$, satisfying $\phi(-\infty) = 0$) to the delayed KPP-Fisher equation

$$u_t(t, x) = \Delta u(t, x) + u(t, x)(1 - u(t - \tau, x)),\ u \geq 0,\ x \in \mathbb{R}^m. \quad (1)$$

First, we show that each semi-wavefront should be either monotone or slowly oscillating. Then a complete solution to the problem of existence of semi-wavefronts is provided. Moreover, we present a short proof of the following natural extension of the famous Wright's 3/2-stability theorem: the conditions $\tau \leq 3/2,\ c \geq 2$ imply the presence of the positive traveling fronts (not necessarily monotone) $u = \phi(x \cdot \nu + ct),\ |\nu| = 1$, in the equation (1).

On cellular automaton and meanfield models of metapopulations

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Éva V.P. Rácz
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Simulations mean an effective tool in studying the spatiotemporal development of ecological systems. In our talk, we present some of our recent experimental results for stochastic cellular automata models of some metapopulations with Wolfram *Mathematica*. After a short summary of the spatio-temporal development of some single-species territory-based models, we consider the competition of several species for territory. In particular, we investigate the occupation strategies of some aggressively spreading species. Simulations give us a lot of information on special properties such as aggregation, diffusion, properties of the boundary of patches; and in general, the role of neighbors in the development. In some cases, simulations can help to improve the meanfield models of the phenomena.

Differential equations with dynamically defined delayed feedback

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The study of some models from population dynamics and epidemiology leads to differential equations where the delay terms are not given explicitly but arise as the solution of another system of differential equations. In this talk we consider initial value problems for differential equations with such dynamically defined delayed feedback. Our goal is to obtain fundamental properties of the system to ensure that the model equations coming from biological applications are meaningful. As an application, we also present an epidemic model for the spread of infectious diseases on long distance travel networks with disease dynamics during travel.

Stabilization of the turning process by using digital feedback control

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In turning processes regenerative chatter limits productivity and the quality of the workpiece. For the increase of the material removal rate without losing the stability of machining digital control strategy is applied. The governing equation of the mechanical system under study is a nonlinear DDE with two delays: one is a point delay, the other one is a piecewise linear time periodic delay.

The stability of the linearised system is discussed. The linear stability is investigated by using two methods. The first method approximates the infinite dimensional problem with a finite dimensional system by using the method of semi-discretization. The second method applies a condition for stability equivalence in order to transform the equation to a DDE with point delays. The results are presented on so-called stability lobe diagrams.

Rational functions with maximal radius of absolute monotonicity

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We study the radius of absolute monotonicity R of rational functions with numerator and denominator of degree s that approximate the exponential function to order p. Such functions arise in the application of implicit s-stage, order p Runge–Kutta methods for initial value problems, and the radius of absolute monotonicity governs the numerical preservation of properties like positivity and maximum-norm contractivity. We construct a function with $p = 2$ and $R > 2s$, disproving a conjecture of van de Griend and Kraaijevanger. We determine the maximum attainable radius for functions in several one-parameter families of rational functions. Moreover, we prove earlier conjectured optimal radii in some families with 2 or 3 parameters via uniqueness arguments for systems of polynomial inequalities. Our results also prove the optimality of some strong stability preserving implicit and singly diagonally implicit Runge–Kutta methods. Whereas previous results in this area were primarily numerical, we give all constants as exact algebraic numbers.

On the logistic equation with two discrete delays

Yukihiko Nakata
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We consider dynamical aspects of logistic equation that has both negative and positive delayed feedback. First we show that positive delayed feedback is responsible for the existence of a series of unbounded solutions. Local and global stability results for a positive steady state are obtained by perturbation technique and oscillation theory of delay differential equations. With analytical results and numerical investigation we discuss asymptotic behavior of solutions with respect to parameter of the equation.

This is a joint work with István Győri and Gergely Röst.

Advanced and delay integro-differential equations in process engineering and insurance mathematics

Éva Orbán-Mihálykó* and Csaba Mihálykó
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In this talk we consider a stochastic model applied in process engineering and insurance mathematics. In these models the main goal is determination of the probability of the reliability and the expected time of ruin. To handle them together, Laplace transform of the density of time of ruin is introduced and an integral equation is derived for it in general case. We prove the existence and uniqueness of the solution of the integral equation in the set of bounded functions and we show that the solution tends to zero exponentially. If the density function of the inter-arrival time satisfies a linear differential equation with constant coefficients, the integral equation is transformed into an integro-differential equation and we provide an explicit solution for it without any further assumption concerning the density function of the filled amount of material.

On the smooth parameter-dependence of the resolvent function

Bernát Slezák
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Using the derivative of the composition (the so called Nemickii-) operator, the following statement is obtained:

If the function f is continuously differentiable then the resolvent function of the equation

$$x'(t) = f(t, x_t), \qquad x_\sigma = \phi$$

is continuously differentiable with respect to the initial parameters (σ, ϕ).

Systems of linear differential equations of second order with constant matrix and constant delay

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We study the applicability of the delayed matrix exponential in order to represent the solution of the Cauchy initial problem for the systems of linear differential equations of second order with constant matrices and for some constant delays. The equivalence of such systems with systems of the first order is used. Due to this equivalence we can obtain asymptotic properties of solutions.

A class of asymptotically stable periodic orbits for strongly monotone maps

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It is well–known that for continuous–time dynamical systems, existence of asymptotically stable periodic orbits is excluded by strong monotonicity. It is also well–known that the analogous result does not hold true in the discrete–time case.

Inspired by A. Simonovits' agent–based model for tax evasion, we present moving average network examples into this latter direction. The approach is based partly on modifications of a spatially discrete dynamics of periodic points with certain monotonicity properties in one dimension, partly on elementary eigenvalue facts about the adjacency matrix of the underlying network.

The talk is based on ongoing research with *Barnabás M. Garay.*